

A COMPARATIVE STUDY OF THE SPECTRA OF α BOÖTIS AND 70 OPHIUCHI A

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ABSTRACT

The weaker lines in the photographic region of the spectra of α Boötis and 70 Ophiuchi A have been compared visually and photometrically on high-dispersion coude plates. A résumé of the behavior of the elements in the two stars is given. The relative weakness of the λ 3883 band of CN and several multiplets of Ca I in the giant and the relative strength of La II and the singly ionized rare earths are especially noted.

Curves of growth for the neutral iron lines have been constructed for both stars. The observed velocity and damping factor for α Boötis are 3.25 km/sec and 0.8×10^{-6} , respectively. The solar values were assumed for the dwarf.

Relative abundances are obtained, and departure from equilibrium is evident for the giant. The Adams-Russell formula provides a satisfactory correction. Values of the excitation temperatures and of the log of the relative electron pressures are 3160° , 4150° , and -3.74 , -1.10 for the giant and dwarf, respectively. The results are compared to the sunspot spectrum, and the suggestion is made of a stratification of elements in the atmosphere of the giant similar to that found in the sun.

I. INTRODUCTION

In her classification of stellar spectra, Miss A. C. Maury¹ called attention to the fact that the sudden decrease of intensity in the spectrum from λ 3889 toward the ultraviolet, as well as the degree of absorption in the regions $\lambda\lambda$ 4055–4078 and $\lambda\lambda$ 4144–4216, differs for various stars within the same spectral classification. J. C. Kapteyn² noted that stars of small proper motion showed greater absorption in the ultraviolet than stars of large proper motion and attributed this differential absorption to interstellar scattering. W. S. Adams³ first suggested that this difference in absorption might be connected with the intrinsic luminosity of the star. However, the outstanding differences in the spectra of giant and dwarf stars of similar spectral type were first shown by Adams and Kohlshütter.⁴ They found that “the continuous spectrum of the small proper motion stars is relatively fainter in the violet as compared with the red than is the spectrum of the large proper motion stars.” From a study of the spectral lines they found “systematic differences of intensity for certain lines between stars of large and stars of small proper motion” and listed the most prominent lines. The lines strengthened in the giant are λ 4077 and λ 4216 of Sr II; whereas λ 4325 of Sc II, $\lambda\lambda$ 4435, 4436, and 4455 of Ca I, and λ 4535 of Ti I are weakened in the giant. The ratios of the intensities of these crucial “absolute-magnitude lines” were then calibrated for stars of known parallax and provided a method for the determination of the absolute magnitude and hence the (spectroscopic) parallax.⁵ Thus it has been possible to determine the parallaxes of stars for which no trigonometric parallaxes are available.

B. Lindblad⁶ showed that the sharp decrease of intensity to the violet of λ 3883 is due not only to the differences in the energy distribution of the continuum of giants and

¹ *Harvard Ann.*, **28**, 38, 1897.

² *Ap. J.*, **29**, 46, 1909; *Mt. W. Contr.*, No. 31.

³ *Ap. J.*, **39**, 89, 1914; *Mt. W. Contr.*, No. 78.

⁴ *Ap. J.*, **40**, 385, 1914; *Mt. W. Contr.*, No. 89.

⁵ Adams, Joy, Strömberg, and Burwell, *Ap. J.*, **53**, 13, 1921; *Mt. W. Contr.*, No. 199.

⁶ *Ap. J.*, **55**, 85, 1922.



dwarfs but also to a strong cyanogen band and that the strong absorption in the giants at λ 4216 and λ 4606 is also due to cyanogen. Thus the basis for the cyanogen discriminant in the spectra of giants and dwarfs was established. W. Luyten⁷ has called attention to the weakening of the sodium D lines in the late-type giants, and Miss C. G. Burwell⁸ has studied the behavior of calcium in the red region of giants and dwarfs of late spectral types. She found that the lines of the neutral atom were weakened in the giants, with the exception of λ 6450, which is strengthened. More recently, Morgan, Keenan, and Miss Kellman⁹ have summarized the criteria for absolute-magnitude classification for the different spectral classes in the photographic region, based on the earlier findings of Adams and Kohlschütter, Lindblad, and E. G. Williams.¹⁰

In general, the criteria for absolute-magnitude classification are adapted for spectra of relatively low dispersion, and on high-dispersion spectrograms many of these outstanding features are lost. The cyanogen band at λ 4216, one of the most important criteria for luminosity classification of low-dispersion spectrograms, shows very little change from dwarf to giant on high-dispersion plates, since the band is resolved into its component lines, and the net effect of the increased absorption is considerably diminished. However, on high-dispersion plates many additional lines appear and many blends are resolved, thus permitting a study of the behavior of many elements in some detail. Adams and Russell¹¹ have studied the behavior of a great number of lines of several dwarf, giant, and supergiant stars on high-dispersion spectrograms and, from their data for the various atoms, have calculated the relative abundances, temperatures, electron pressures, etc., with the sun as reference. In the present investigation, high-dispersion spectra of the giant and dwarf K0 stars— α Boötis and 70 Ophiuchi A—have been compared, with emphasis on the behavior of the fainter lines in the photographic region.

The star α Boötis is classified as K0 in the Draper Catalogue, at Victoria, and at Mount Wilson. The Yerkes classification is "slightly earlier than the mean for class gK2" and is given as gK2 pec. The trigonometric parallax¹² of the star is $+0''.080 \pm 0''.005$, while the spectroscopic parallaxes determined at Mount Wilson, Victoria, and Harvard are $0''.10$, $0''.10$, and $0''.21$, respectively.¹³ The discrepancy has been attributed to the high-luminosity value obtained from the relative intensity of λ 4077 of ionized strontium to the neighboring iron lines, and the abnormally weak cyanogen band to the violet of λ 4215.¹⁴ Pettit and Nicholson¹⁵ have determined the surface temperature as 4300° ; the atmospheric temperature determined by Adams and Russell¹⁶ is 3220° (referred to the solar value of 4700°). Interferometer measurements of the diameter by F. G. Pease¹⁷ indicate a value $0''.020$. Combined with the trigonometric parallax, this gives a radius twenty-seven times that of the sun. The mass is of the order of nine times that of the sun, and the radial velocity determined by Adams¹⁸ is -5.621 ± 0.005 km/sec.

The system of 70 Ophiuchi consists of two dwarf K stars. The brighter component is classified K1 in the Draper Catalogue and K0 at Victoria, Mount Wilson, and Yerkes. If Kuiper's¹⁹ values for the apparent visual magnitude, $+4.19$, and the parallax, $0''.196$

⁷ *Pub. A.S.P.*, **35**, 175, 1923.

⁸ *Pub. A.S.P.*, **42**, 352, 1930.

⁹ *An Atlas of Stellar Spectra*, University of Chicago Press, 1943.

¹⁰ *A.p. J.*, **83**, 305, 1936.

¹¹ *A.p. J.*, **68**, 9, 1928.

¹² *General Catalogue of Stellar Parallaxes*, New Haven, 1924.

¹³ Russell, Dugan, and Stewart, *Astronomy*, **2**, 875, 1938; *Mt. W. Contr.*, No. 551.

¹⁴ Morgan, Keenan, and Kellman, *op. cit.*

¹⁷ *Pub. A.S.P.*, **57**, 28, 1945.

¹⁵ *A.p. J.*, **68**, 279, 1928.

¹⁸ *A.p. J.*, **90**, 494, 1939; *Mt. W. Contr.*, No. 638.

¹⁶ *Loc. cit.*

¹⁹ *A.p. J.*, **95**, 205, 1942.

$\pm 0''.004$, are adopted, the absolute magnitude of the star is $+5.7$. The fainter component is K5, with an absolute magnitude $+7.3$. P. van de Kamp²⁰ gives a period of 87.8 years, semimajor axis of 23.3 astronomical units, and masses 0.90 and 0.73 that of the sun for the A and B components, respectively. The temperature of the brighter component is 5080° according to measures of heat-index and water-cell absorption by Pettit and Nicholson.²¹

II. DATA AND MEASUREMENTS

The plates used in the investigation were taken in the second order of the grating spectrograph at the coudé focus of the 100-inch telescope. The focal length of the camera is 114 inches, giving a dispersion of 2.9 Å per millimeter. The microphotometric tracings were obtained with the Mount Wilson microphotometer at a magnification of 60.

In a preliminary survey the spectra of α Boötis and 70 Ophiuchi were compared visually on the Hartmann spectrocomparator over the entire wave-length range, and a visual scale of apparent difference of intensity from giant to dwarf was established. However, a calibration of this scale in terms of a difference of equivalent width indicated considerable scatter. The intensity scale was affected to a large extent by the complex variation in the differences of the background densities of the plates and by the degree of blending of the individual lines and, to a slighter degree, by the number of lines in the field. Accordingly, a combination of visual and photometric data was used. The spectra were re-examined visually with the corresponding high-magnification microphotometric tracings, and a new visual scale was established. In this case an attempt was made visually to account for the factors mentioned above, and subsequent calibration disclosed a remarkable consistency in the visual scale over the entire wave-length range. Photometric estimates were then made of the equivalent widths of all lines in the two stars, which apparently differed in intensity in the visual survey. These estimates were based on data obtained on similar plates and tracings of Venus for 100 relatively unblended lines in four wave-length regions in the $\lambda\lambda$ 3900–4808 interval. A double-entry table was formed, giving the equivalent width in angstrom units as a function of the central intensity and b , the width of the line at the continuum, measured in millimeters on the tracing. Equivalent widths measured by Allen²² and those measured by the author in the *Photometric Atlas of the Solar Spectrum*²³ were used in the calibration. The probable error for an unblended line is ± 0.005 for an equivalent width of 0.035 Å and ± 0.009 for 0.100 Å. There is, of course, a large margin for error in the measurement of b , even for an unblended line; and the error increases as a function of the background density of the plate. In the case of physical and optical blends, the error is further increased because only a most probable value of b can be used. Thus the estimated equivalent width is a measure of the observed absorption—at a given wave length—which is assumed to have the true continuum as background. It should not be confused with the actual value of the equivalent width, which is a measure of the number of atoms effective in the production of a given atomic or molecular transition. The estimated value will be most seriously in error in this respect for the blend of a faint line near the core of a very strong line. It will be a measure of the number of effective atoms in the case of an unblended line arising from the undisturbed continuum, in which case the error will be solely one of measurement. The advantage of the procedure lies in the fact that they are obtained in a fraction of the time required for the measurement of accurate equivalent widths and yet make possible a preliminary quantitative discussion of the behavior of a great number of lines of various atoms.

For the great majority of lines a mean value of the differential absorption as deter-

²⁰ *Pub. A.S.P.*, **57**, 38, 1945.

²¹ *Op. cit.*

²² *Mem. Commonwealth Obs.*, **1**, Part IV, 2, 1934.

²³ Minnaert, Mulders, and Houtgast, Amsterdam, 1940.

mined visually and photometrically was used. In cases of seriously blended lines, the visual estimate was favored in a weighted mean. To calibrate the visual scale, the differential photometric values were plotted against the visual estimates for groups of some fifty lines, covering, on the average, forty angstrom intervals. The resulting relationships were examined in sections as functions of the background density and *in toto* as functions of the degree of blending and of the strength of the lines. No systematic trends were found. However, there appeared a consistent overestimate of the increased absorption in the spectrum of 70 Ophiuchi, corresponding to one step on the visual scale of ten. This was probably due to the consistently weaker plates of the star and to the difference in the appearance of the two spectra. The latter was due, in turn, to a difference in guiding in obtaining the two spectra.

The identification of the lines was simplified by the use of the *Solar Atlas* and the latest revision of the Rowland Table. The tracings of the two stars were of magnification very similar to that of the *Atlas*. All the lines appearing on the tracings were identified, except for a very few for which the temperature effect was sufficiently great to obliterate the line in the *Atlas*.

A total of 1045 lines within the range of Rowland intensity -1 to $+4$ were found to differ in intensity between α Boötis and 70 Ophiuchi A in the wave-length interval 3900–4808. Of these, 195 lines are unidentified in the sun. The complete list of lines is too extensive to include in the present paper. However, the lines for which no solar identifications are known are of particular interest and have been included in the Appendix. The data for the section $\lambda\lambda$ 4265–4318 have been omitted because of possible errors due to slight irregularities at the cut edges of the plates. The lines are listed in the Appendix according to wave length, with disk and spot intensities from the Revised Rowland Table. The fourth column gives a measure of the relative increase of intensity between the two stars. The star in which the line appears stronger is denoted by “d” for “dwarf,” i.e., 70 Ophiuchi A, and by “g” for “giant,” i.e., α Boötis. An intensity difference of approximately 10–25 mÅ is denoted by 1; a difference of 25–40 mÅ by 2; a difference of 40–65 mÅ by 3; and a difference of 65–100 mÅ by 4. The lines have been classified in the fifth column, using the differential absorption between the disk and the spot spectra and between the dwarf and the giant spectra. Class A lines are those for which the increase or decrease in intensity from dwarf to giant is in the same direction as from disk to spot. Class B lines are those for which no change in intensity is given from disk to spot. Class C lines are those for which the change in intensity from dwarf to giant is reversed as from disk to spot. No classification is given to those lines for which no spot intensities are given.

In general, class A lines indicate chiefly temperature effects, class C lines are sensitive to pressure effects, and class B lines will be a combination of the two effects. For lines which strengthen in the giant in classes B and C, we can suspect *La* II, the singly ionized rare earths, faint compounds, and possibly *Sr* II and *Ba* II. For the lines which strengthen in the dwarf in classes B and C, we can suspect *Ca* I, *Na* I, *Mn* I, and possibly faint *CH*.

The fainter of the two stellar lines will, in general, be of the same order of magnitude as the solar line. Information regarding the structure and the degree of blending can thus be obtained to some extent from the *Solar Atlas*. For all faint lines of intensity difference 1, however, the classification of lines may be quite arbitrary, since the change in intensity for the stellar and solar lines may depend heavily on the background blending.

III. BEHAVIOR OF ELEMENTS

In the following section a résumé of the behavior of the various atoms is given, as shown by the weaker lines. *H*, *Ca* II, *CN*, and *CH* are included for the sake of completeness. In the descriptions the expression “unresolved line” refers to a wave length for which more than one identification is given. A “resolved line” in this manner refers to a

wave length which is attributed uniquely to a given atomic or molecular transition, regardless of the degree of physical or optical blending in the spectrum. The terms "giant" and "dwarf" are used for simplicity, although the results refer specifically to *a* Boötis and 70 Ophiuchi A.

- H* In the dwarf the lines have strong wings, whereas in the giants they are quite sharp.
- Na* I Only three classified lines occur in the region. The only resolved line (E.P. 2.1) is stronger in the dwarf. The behavior of the line is attributed to absolute-magnitude effect.
- Mg* I The single line arising from the lowest level, 3^1S , is enhanced in the giant. Lines arising from the 3^1P^o level (E.P. 4.3) are stronger in the dwarf.
- K* I The two classified lines occurring in the region arise from the lowest level, 4^2S (E.P. 0.0), and are strengthened in the giant.
- Ca* I All lines observed are stronger in the dwarf with the exception of those involved in the $4^3P^o - p^2\ ^3P$ transition (E.P. 1.9). These lines are stronger in the giant. The decrease in the intensity of the majority of the lines from dwarf to giant is one of the outstanding absolute-magnitude effects.
- Ca* II The H and K lines are the strongest lines in both spectra. The double reversal of the lines is considerably more pronounced in the giant.
- Sc* I Lines arising from the a^2D and a^4F levels (E.P. 0.0, 1.4) are considerably strengthened in the giant.
- Sc* II All lines in the region arise from levels with E.P. less than 0.7, with the exception of one which arises from the b^1D level (E.P. 1.4). All are strengthened in the giant.
- Ti* I All lines are considerably strengthened in the giant (E.P. 0.0-2.4).
- Ti* II All lines are strengthened in the giant (E.P. 0.6-2.0).
- V* I All lines are strengthened in the giant (E.P. 0.0-2.1).
- V* II All lines are strengthened in the giant (E.P. 1.5-2.8).
- Cr* I All lines arising from levels with E.P. less than 3.2 are strengthened in the giant. At somewhat higher values the lines are strengthened in the dwarf.
- Cr* II The lines arising from the b^4D level (E.P. 3.1) are slightly strengthened in the giant. Those arising from the b^4F level (E.P. 4.1) are strengthened in the dwarf.
- Mn* I All the fainter lines occurring in the region are of E.P. 2.2 or greater and are strengthened in the dwarf.
- Mn* II No unresolved line was observed in the region and only one resolved line, which is of unknown classification. The behavior of the atom is indeterminate.
- Fe* I All lines arising from levels with E.P. less than 3.1 are strengthened in the giant, while those arising from higher levels are strengthened in the dwarf. The z^5D^o level may be somewhat enhanced in both stars. Lines arising from the lowest b levels in the giant are slightly stronger than those arising from the a and z levels of a corresponding E.P. Several multiplets of high E.P. are considerably winged in the dwarf.
- Fe* II Only the b^4P and b^4F multiplets (E.P. 2.6, 2.8) occur in the region. Both are strengthened in the dwarf.

- Co* I All the lines recorded are strengthened in the giant (E.P. 0.0–3.2).
- Ni* I Lines arising from levels of E.P. 0.0–1.9 are strengthened in the giant. Lines arising from levels with E.P. greater than 3.3 are somewhat strengthened in the dwarf.
- Ni* II Only one unresolved line was found, and it was slightly strengthened in the giant. The behavior of the atom is indeterminate.
- Zn* I The three classified lines occurring in the region are all of high E.P. The line λ 4722 (E.P. 4.0) is slightly strengthened in the dwarf.
- Ge* I Two classified lines occur in the region, one of which, λ 4686 (E.P. 2.0), is strengthened in the giant.
- Sr* I Of the three classified lines occurring in the region, one was strengthened in the giant.
- Sr* II The two ultimate lines (E.P. 0.0) in the region, λ 4078 and λ 4216, $5^2S-5^2P^o$, are strengthened in the giant. The intensities of these lines are often used for the classification of giants and dwarfs.
- Y* I All lines occurring in the region are weak. Those arising from the a^2D level (E.P. 0.0) are considerably strengthened in the giant.
- Y* II Of the ten unresolved classified lines in the region, one was observed stronger in the giant. The behavior of the atom is indeterminate.
- Zr* I The lines occurring in the region all arise from low-energy levels and are strengthened in the giant.
- Zr* II The a^2D and a^2F levels (E.P. 0.5, 0.7) are strengthened in the giant. Lines arising from higher levels show no change in intensity.
- Cb* I All lines occurring in the region are strongly blended or of questioned identification. The behavior of two unresolved lines suggests a strengthening in the giant.
- Ru* I The six classified lines occurring in the region are all very faint. The line λ 4081 is strengthened in the giant.
- Rh* I Only one resolved and unclassified line was noted in the region. The behavior of the atom is indeterminate.
- Ba* II The only line occurring in the region which arises from the lowest level, 6^2S (E.P. 0.0), λ 4554, is considerably strengthened in the giant. This line is particularly sensitive to absolute magnitude.²⁴ Lines belonging to the higher-energy levels (E.P. 2.5–2.7) are unresolved and probably remain constant.
- La* II All lines observed from E.P. 0.0 to E.P. 1.9 are strengthened in the giant. It is noted that in the spectra of the disk and spot no change of intensity is found.²⁵ The increase in the giant is probably due to absolute-magnitude effect.
- Ce* II, *Pr* II, *Nd* II, *Sm* II, *Eu* II, *Gd* II, *Dy* II, *Lu* II The rare earths are all considerably strengthened in the giant (E.P. 0.0–0.8). It is noted that the lines of these atoms remain unchanged from disk to spot.²⁶ Europium and then neodymium show the greatest change in intensity. The strengthening is probably due to absolute magnitude unless α Boötis is characterized by unusually strong lines of these atoms.

²⁴ Morgan, Keenan, and Kellman, *op. cit.*

²⁵ C. E. Moore, *Ap. J.*, **75**, 304, 1932.

²⁶ *Ibid.*

- W* I There are no resolved classified lines in the region, but there is an indication that two unresolved lines may be strengthened in the giant.
- Pb* I The only line occurring in the region is strengthened in the giant (E.P. 1.3).
- CH* The $2\Delta - 2\Pi$ band at λ 4300 is strengthened in the dwarf.
- CN* The $2\Sigma - 2\Sigma$ (0, 0) band at λ 3883 was observed visually and is somewhat strengthened in the dwarf. The $2\Sigma - 2\Sigma$ (0, 1), (1, 2), (2, 3) bands are strengthened in the giant.

IV. QUANTITATIVE ANALYSIS OF THE ATOMIC SPECTRA

A. CURVES OF GROWTH FROM THE IRON SPECTRUM

1. *General considerations.*—The most complete set of data for a study of the abundance, temperature, pressure, and departures from equilibrium is afforded by the series of iron lines. The estimated equivalent widths of the resolved lines were used to determine curves of growth for α Boötis and 70 Ophiuchi. The curves were then used for the determinations of the relative abundances of the atoms other than neutral iron at different excitation potentials. From these data the temperatures and electron pressures were determined.

K. O. Wright²⁷ has determined an empirical curve of growth for the sun, giving $\log W/\lambda$ as a function of $\log X_{f(ab)}$, where

$$\log X_{f(ab)} = \log gf + \log \lambda - \frac{5040E}{T} = \log X_0 - 1.81 \quad (1)$$

and gf is the laboratory intensity of the line. ($\log X_0$ is defined by Menzel²⁸ for a particular model of radiative transfer.) Let X_r be defined as follows:

$$\log X_r = 5040E \left(\frac{1}{T_\odot} - \frac{1}{T_*} \right) + \log X_\odot. \quad (2)$$

The best excitation temperature of the star will be determined by the slope of the straight line relating $\log X_r/X_\odot$ to the excitation potential. $\log X_r/X_\odot$ is determined by the shifts in $\log X$ needed to eliminate the systematic displacement of multiplets of different excitation potentials. These displacements result from a difference in temperature between the sun and the star. If the stellar equivalent widths are accurate, it is possible to construct the curve of growth directly, commencing in the manner described. However, the estimated values are but approximations and may be systematically high or low. This would lead to serious error in building up the final curve of growth, especially for a giant star whose absorbing atoms differ considerably in kinetic velocity from those of a dwarf. The effect of the velocity is inherent in the curve of growth of the star; and in practice it shows up as a vertical displacement in the $\log W/\lambda$ co-ordinate.

2. *Application to α Boötis.*—To establish absolute values for the curve of growth of α Boötis, 53 strong and comparatively unblended iron lines were measured on the direct-intensity tracings by Williams and Hiltner²⁹ in the red region of the spectrum. Eight lines arising from the a^5F level ($\overline{E.P.}$ 0.97) and 10 lines arising from the a^3F level ($\overline{E.P.}$ 1.55) were sufficiently strong to assume pure damping broadening with a curve of growth of slope $\frac{1}{2}$. $\log W/\lambda$ was computed for these lines in α Boötis and in the sun. The equivalent widths of the solar lines given by Allen³⁰ were then assumed to fall exactly on the empirical curve of growth for the sun given by Wright.

²⁷ *Ap. J.*, **99**, 249, 1944.

²⁸ *Ap. J.*, **84**, 462, 1938.

²⁹ Unpublished data from coudé spectrograms obtained at the McDonald Observatory.

³⁰ *Op. cit.*

The very strong lines of the a^5F and a^3F multiplets in α Boötis were then made to coincide with the straight-line portion of the curve of growth of the sun by horizontal displacements in $\log X_\odot$, giving $\log X'_\odot$, let us say. The estimated values of the weak lines of these multiplets were then shifted by the corresponding amounts. A tentative straight-line relationship was assumed between the excitation potential and $\log X'_\odot/X_\odot$, and the shift in $\log X'_\odot/X_\odot$ was then extrapolated for the a^5D level (E.P. 0.0) and the a^3P , a^5P levels (E.P. 2.3). This was applied to both sets of data, and thus a curve of growth was tentatively constructed. The estimated values fell consistently low in the $\log W/\lambda$ coordinate, but the different multiplets fitted together consistently. The lines arising from the z^5D^o level (E.P. 3.2) and the z^5P^o level (E.P. 3.7) were then added and the best fit obtained.

The curve determined by the estimated values fell systematically lower in $\log W/\lambda$ than the direct-intensity curve, although similar slopes are indicated. The curves relating $\Delta \log X_\odot$ to the excitation potential fitted closely for the two sets of data. Thus the former curve was corrected for this systematic difference, amounting to 30 per cent of the estimated equivalent widths, and a common curve was derived for both sets of data. This systematic error for α Boötis was probably due to an underestimate of the continuum of the spectrum of the star.

Actually, there is no need to shift the very strong lines of the various multiplets through $\Delta \log X_\odot$. In this particular instance it was done to enable an extrapolation of the equivalent widths of the lines of the a^5D multiplet which were not available by direct measurement. If $+1.38$ represents $\Delta \log X_\odot$ at E.P. = 0.0, then from equation (2) we have

$$\log \frac{X'_\odot}{X_\odot} - 1.38 = 5040E \left(\frac{1}{T_\odot} - \frac{1}{T_*} \right) = \log \frac{X_r}{X_\odot}. \quad (3)$$

The curve relating $\log W/\lambda$ to $\log X_r$ was fitted to the family of theoretical curves of growth given by Menzel.³¹ These curves give $\log (c/v \cdot W/\lambda)$ as a function of $\log Z$, where

$$Z = \frac{\Gamma}{\nu_0} \frac{c}{v}.$$

Thus the vertical coincidence of the empirical and theoretical curves yields the velocity of the atoms in the star, v_* ; and the horizontal coincidence gives Γ/ν_0 , the damping factor; and $\log X_r - \log X_*$. The velocity and the damping factor for the star are 3.25 km/sec and 0.8×10^{-6} , respectively. The apparent velocity well exceeds that which is due solely to kinetic motion. Thus the observed value must be attributed to the combined effects of kinetic and turbulent velocities. The curves of growth for α Boötis are given in Figures 1, 2, and 3. In Figure 3 the smooth curve represents the theoretical one. The empirical curve lies above the theoretical curve where the Doppler region changes to the transition portion; and beyond $\log X_* = +1.3$ it lies below the theoretical curve until the transition portion changes to the damping region. This same effect was noted to a lesser degree by Wright in his solar curve and is apparent in the curve for Sirius determined by Aller.³² A large proportion of the departure from the theoretical curve in the present data may be due to the uncertainty in the values for the Doppler region. A small change in the horizontal displacement of the initial rise of the curve would change the length of the transition portion, and hence the damping factor. Writing the equations in X for the same line in the sun and a star and dividing, we obtain

$$\frac{X_*}{X_\odot} = \frac{N_{a*}}{N_{a\odot}} \frac{b(T_\odot)}{b(T_*)} e^{-E_i(1/T_* - 1/T_\odot)} \frac{v_\odot}{v_*}. \quad (4)$$

³¹ *Pop. Astr.*, **47**, 74, 1939.

³² *Ap. J.*, **96**, 328, 1942.

Therefore,

$$\log \frac{X_*}{X_r} = \log \frac{N_{a*}}{N_{a\odot}} + \log \frac{b(T_{\odot})}{b(T_*)} + \log \frac{v_{\odot}}{v_*}. \quad (5)$$

Since $\log X_*/X_r$ and $\log v_*/v_{\odot}$ are now known, $\log N_{a*}/N_{a\odot}$ is directly obtained. Furthermore, since for a given line in the sun and the star

$$\log \frac{X_*}{X_{\odot}} + \log \frac{v_*}{v_{\odot}} = \log \frac{N_*}{N_{\odot}}, \quad (6)$$

we are now in a position to obtain the relative number of atoms in the two stars for the lines of any element at any given atomic level.

3. *Application to 70 Ophiuchi.*—Only a few very strong lines are available in the photographic region for the determination of absolute values for the curve of growth for

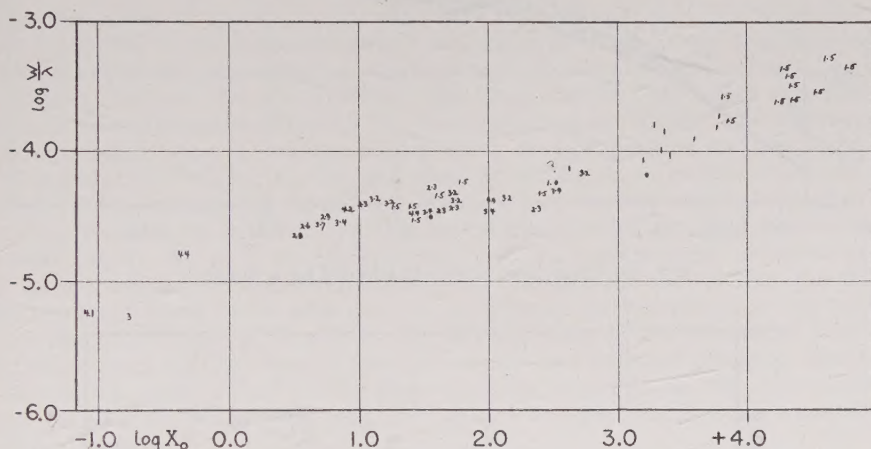


FIG. 1.—Curve of growth for the neutral iron lines in α Boötis from direct-intensity measures

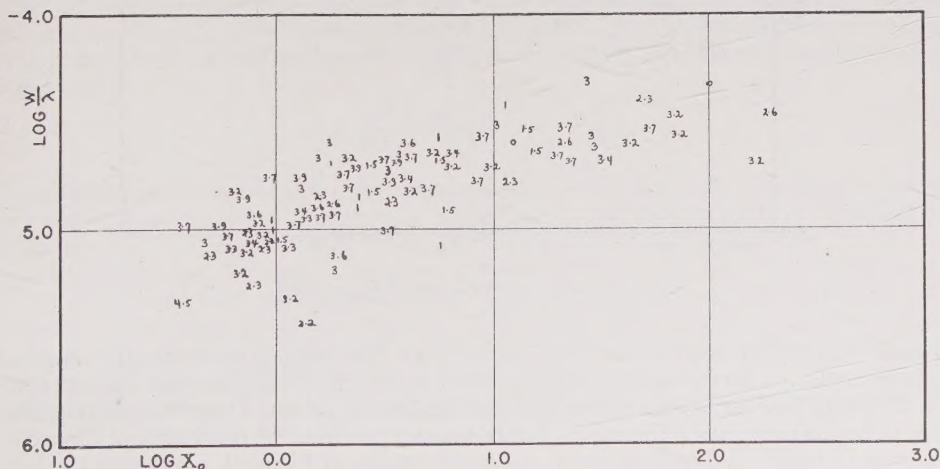


FIG. 2.—Curve of growth for the neutral iron lines in α Boötis from estimated values

70 Ophiuchi. Thus the estimated values of the equivalent widths were used to determine the curve. It was assumed (*a*) that the mean curve drawn through the scattered points maintained the same shape as the solar curve and (*b*) that $\log X_*/X_\odot$ decreased linearly with increasing excitation potential. The first assumption is admissible, since we are considering a star quite comparable to the sun, and we would not expect any radical change in the turbulent velocities. The second assumption is equally admissible, since

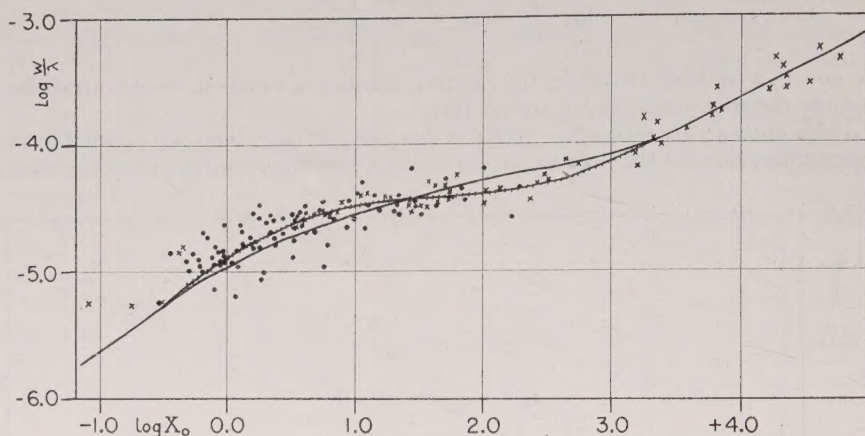


FIG. 3.—Final curve of growth adopted for α Boötis

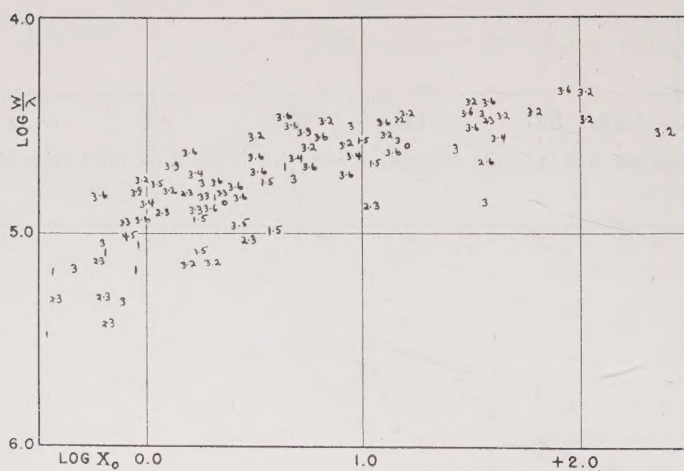


FIG. 4.—Curve of growth for 70 Ophiuchi A

we know that the K0 star is cooler than the G0 star. The assumption of linearity implies that the conditions for thermodynamic equilibrium are fulfilled in a manner similar to the sun. This may well serve for a preliminary adjustment of values. These two assumptions serve to fix the curve along the $\log X_0$ axis but do not preclude the possibility of a systematic error in $\log W/\lambda$. The final curve of growth, shown in Figure 4, represents the best over-all adjustment of values for different excitation potentials. This was obtained by

successive approximations for the relative shifts of the individual multiplets which would yield the best straight line relating $\log X_*/X_\odot$ to the excitation potential. Thus the individual values of $\log X_*/X_\odot$ should not be considered purely empirical; rather, the whole assemblage of values represents the best empirical determination.

Two independent checks on the reliability of the curve for 70 Ophiuchi are available. Five lines arising from the a^5F multiplet and falling on the pure damping portion of the curve are available in the photographic region. These were used to determine $\log X_*/X_\odot$ at $\bar{E.P.} = 0.99$. The results agreed closely with the estimated values. Also we should expect the rare earths to remain of similar intensity in the K0 and G0 dwarfs, and thus the values of $\log X_*/X_\odot$ should remain close to 0.00. We shall see later that the mean value derived is +0.10. Thus we may assume that the error in the empirical curve is not so great as to affect the analysis seriously.

B. RELATIVE ABUNDANCE OF VARIOUS ATOMS

With the curves of growth for the estimated equivalent widths of the neutral iron lines in both stars, it was possible to obtain values of $\log N_*/N_\odot$ for the remaining atoms. The same preliminary procedure was followed as in the case of the iron lines. $\log W/\lambda$ was computed for both the solar and the stellar lines, and the solar values were assumed to fall exactly on the curve of growth for the sun as given by Wright. The differences in $\log X$ required to bring the various multiplets to the best position on their respective curves of growth is then $\log X_*/X_\odot$, and when we add $\log v_*/v_\odot$ to this we immediately have $\log N_*/N_\odot$. In obtaining the data for 70 Ophiuchi, it was assumed that $\log v_*/v_\odot = 0$. The data are summarized in Table 1, giving the mean excitation potential, the number of lines used, $\log N_*/N_\odot = Y$ for the two stars, and the spectroscopic notations of the lower levels included in each determination. In the data for neutral iron, the starred values represent those which were used in the original determinations of the curve of growth. The other values of $Fe\ I$ were originally interpolated or extrapolated from Figure 5, which gives $\log X_\odot'/X_\odot$ as a function of the low excitation potential. The lower line of numbers under "No." of $Fe\ I$ gives the lines measured on the direct-intensity tracings for α Boötis. Only those lines known to have a single identification were used in compiling the table.

C. DETERMINATION OF TEMPERATURES

It was pointed out earlier that "the best excitation temperature (relative to the sun) will be determined by the slope of the line relating $\log X_*/X_\odot$ to the excitation potential." If the conditions for thermodynamic equilibrium are fulfilled, this relationship will be linear and

$$S' = \frac{\Delta Y}{\Delta E} = 5040 \left(\frac{1}{T_\odot} - \frac{1}{T_*} \right). \quad (7)$$

This relationship appears to be quite satisfactory for 70 Ophiuchi. In the case of α Boötis, however, it is not possible to assume a simple linear relationship, and the situation becomes a trifle puzzling. It is hard to see which point on the curve should be used for the slope determination. We could determine the slope at $E = 0$ as well as at a high excitation potential and treat these slopes as the values for the minimum and maximum temperatures. From Figure 5 we find these to be 3200° and 3950° , respectively, when we use the solar value by Wright³³ of 4900° for $Fe\ I$. The maximum temperature was calculated at an excitation potential of 3.0 volts.

Adams and Russell³⁴ found that the departure from linearity was very marked for

³³ *Op. cit.*, p. 250.

³⁴ *Op. cit.*, p. 23.

TABLE 1
OBSERVED VALUES OF THE RELATIVE ABUNDANCES $Y = \log N_*/N_\odot$

Element	$\overline{E.P.}$	No.	Y_ϵ	Multiplet	Y_d
<i>Fe I</i>	0.00	$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$	+1.94	$\begin{Bmatrix} a^3D \\ a^5D \end{Bmatrix}$	+0.60
	0.97	$\begin{Bmatrix} 8 \\ 10 \end{Bmatrix}$	1.47*	$\begin{Bmatrix} a^5F \\ a^5F \end{Bmatrix}$	+ .42*
	1.55	$\begin{Bmatrix} 8 \\ 16 \end{Bmatrix}$	1.12*	$\begin{Bmatrix} a^3F \\ a^3F \end{Bmatrix}$	+ .35*
	2.27	$\begin{Bmatrix} 8 \\ 4 \end{Bmatrix}$	0.90*	$\begin{Bmatrix} a^3P, a^5P, a^3H \\ a^3P, a^5P, a^3H \end{Bmatrix}$	+ .26*
	2.58	$\begin{Bmatrix} 3 \\ 1 \end{Bmatrix}$	0.78	b^3F	+ .23
	2.87	$\begin{Bmatrix} 5 \\ 2 \end{Bmatrix}$	0.68	z^7F^0, z^7P^0	+ .18
	2.99	$\begin{Bmatrix} 6 \\ 1 \end{Bmatrix}$	0.66	b^3G	+ .17
	3.24	$\begin{Bmatrix} 20 \\ 6 \end{Bmatrix}$	0.56*	z^5D^0	+ .15*
	3.27	$\begin{Bmatrix} 5 \\ 5 \end{Bmatrix}$	0.54	a^3D	+ .14
	3.39	$\begin{Bmatrix} 5 \\ 4 \end{Bmatrix}$	0.53	z^5F^0	+ .13
	3.56	$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$	0.51	a^4H	+ .10
	3.62	$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$	0.49	b^3D	+ .09
	3.67	$\begin{Bmatrix} 18 \\ 1 \end{Bmatrix}$	0.56*	z^5P^0	+ .10*
	3.90	$\begin{Bmatrix} 6 \\ 6 \end{Bmatrix}$	0.41	z^3F^0, z^3D^0	+ .07
	2.66	$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$	0.11	a^4H	- .18
	2.77	$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$	0.09	b^4P	- .19
	2.83	$\begin{Bmatrix} 9 \\ 9 \end{Bmatrix}$	0.06	b^4F	- .20
	0.03	$\begin{Bmatrix} 6 \\ 6 \end{Bmatrix}$	2.51	a^3F	+ .65
<i>Ti I</i>	0.94	$\begin{Bmatrix} 15 \\ 15 \end{Bmatrix}$	1.76	a^3F, a^3P, a^1D	+ .65
	1.80	$\begin{Bmatrix} 19 \\ 19 \end{Bmatrix}$	1.61	a^3P, a^3G	+ .40
	2.10	$\begin{Bmatrix} 12 \\ 12 \end{Bmatrix}$	1.46	z^5G^0, z^5F^0	+ .50
	2.17	$\begin{Bmatrix} 13 \\ 13 \end{Bmatrix}$	1.71	a^3H, a^3D	+ .50
	2.33	$\begin{Bmatrix} 5 \\ 5 \end{Bmatrix}$	1.46	z^5D^0, z^3F^0	+ .45
<i>Ti II</i>	1.12	$\begin{Bmatrix} 13 \\ 13 \end{Bmatrix}$	1.01	a^2G, a^2D, a^4P	+ .17
	1.23	$\begin{Bmatrix} 14 \\ 14 \end{Bmatrix}$	1.01	a^2P, b^4F	+ .07
<i>V I</i>	0.03	$\begin{Bmatrix} 9 \\ 9 \end{Bmatrix}$	2.34	a^4F	+ .60
	0.28	$\begin{Bmatrix} 10 \\ 10 \end{Bmatrix}$	2.16	a^4D	+ .50
	1.10	$\begin{Bmatrix} 9 \\ 9 \end{Bmatrix}$	1.86	a^4D, a^4P	+ .28
	1.99	$\begin{Bmatrix} 8 \\ 8 \end{Bmatrix}$	1.46	a^4H, b^4F, a^4G, z^6G^0	+ .25
<i>V II</i>	1.84	$\begin{Bmatrix} 7 \\ 7 \end{Bmatrix}$	1.71	a^3G, a^3P, b^3G	+ .25
<i>Cr I</i>	0.96	$\begin{Bmatrix} 3 \\ 3 \end{Bmatrix}$	2.01	a^3D, a^3S	+ .65
	2.79	$\begin{Bmatrix} 20 \\ 20 \end{Bmatrix}$	1.21	$a^3G, a^3P, a^3P, z^7P^0, b^5D$	+ .35
	3.23	$\begin{Bmatrix} 3 \\ 3 \end{Bmatrix}$	0.56	z^7F^0, b^3P	+ .12
<i>Cr II</i>	4.06	$\begin{Bmatrix} 3 \\ 3 \end{Bmatrix}$	0.11	b^4F	- .30
<i>Ni I</i>	0.20	$\begin{Bmatrix} 3 \\ 3 \end{Bmatrix}$	1.81	a^1D, a^3F, a^3D	+ .33
	1.67	$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$	0.81	b^1D	- .05
	1.94	$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$	1.06	a^3P	+ .15
	3.51	$\begin{Bmatrix} 7 \\ 7 \end{Bmatrix}$	0.31	z^5F^0, z^5D^0, z^5G^0	- .20
	3.70	$\begin{Bmatrix} 11 \\ 11 \end{Bmatrix}$	0.46	$z^3F^0, z^3P^0, z^3D^0, z^1F^0$	+ .07
<i>Co I</i>	4.08	$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix}$	0.71	y^3F^0	+ .45
	0.12	$\begin{Bmatrix} 6 \\ 6 \end{Bmatrix}$	1.96	a^4F	+ .45
	0.50	$\begin{Bmatrix} 7 \\ 7 \end{Bmatrix}$	+1.69	b^4F	+0.35

TABLE 1—*Continued*

Element	$\overline{E.P.}$	No.	Y_{κ}	Multiplet	Y_d
	0.92	1	+1.86	a^2F	+0.27
	1.94	3	1.26	b^4P	+ .15
	2.71	1	1.08	b^2D	+ .30
	3.13	2	1.06	z^6D^0	+ .25
$Sc\ I$	0.01	6	1.91	a^2D	+ .67
	1.43	3	1.21	a^4F	+ .40
$Sc\ II$	0.53	4	1.23	a^1D, a^3F	+ .07
$Zr\ I$	0.04	2	1.56	a^3F	+ .45
	0.64	8	1.71	a^3P, a^1D, a^3F	+ .70
$Zr\ II$	0.67	3	0.86	a^2D, a^2F	+ .10
$Mn\ I$	2.94	14	0.71	a^4D, z^6P^0	+ .10
$Ca\ I$	2.50	4	0.16	$3^1D, 4^1P^0, 4^3P^0, 3^3D$	+ .18
$La\ II$	0.23	12	1.01	a^3D, a^3F, a^1D	+ .10
$Nd\ II$	0.15	10	1.51	a^6I, a^4I	+ .00
	0.55	3	1.01	a^6I, a^4I	+ .15
$Pr\ II$	0.12	2	1.48	a^8I^0	+ .40
$Sm\ II$	0.20	12	1.28	a^8I^0, a^6F	+ .22
	0.46	9	1.30	a^8F, a^6F	+ .00
$Ce\ II$	0.17	10	0.96	$a^4H, b^4H^0, a^4H^0, a^2H, a^2G^0$	+ .05
	0.61	9	1.30	$b^4H^0, c^4G^0, a^4F, a^2H^0, a^2F^0$	+ .15
$Eu\ II$	0.07	3	+1.66	a^7S^0, a^8S^0	+0.00

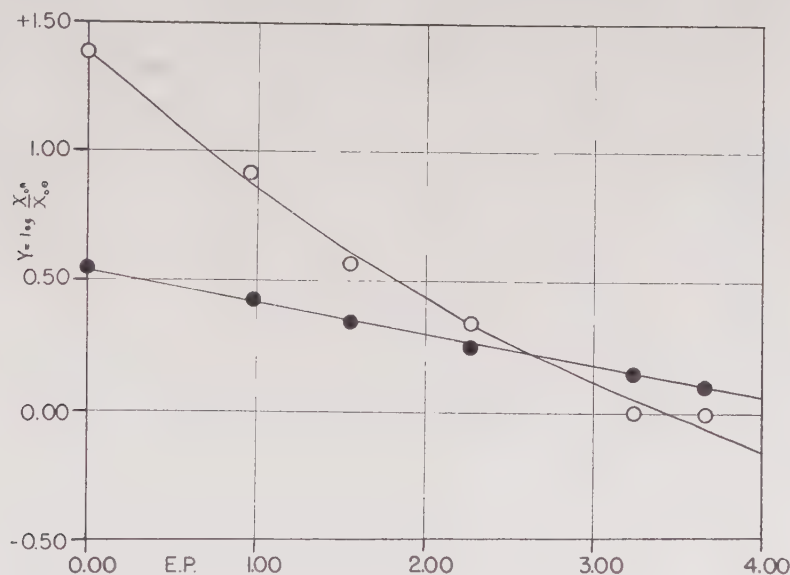


FIG. 5

α Orionis and α Scorpii and evident for α Boötis as well. They found that the curve relating $\log N_*/N_{\odot}$ to E for the two class-M giants could be represented by

$$Y = \log \frac{N_*}{N_{\odot}} = A - 1.05E + 0.087E^2. \quad (8)$$

A is the intercept on the axis of E for the particular curve considered. They then assumed more generally that the observed quantity Y was a linear function not of E but of a quantity X defined by

$$X = 1.05E - 0.087E^2 \quad (9)$$

such that

$$Y = Y_0 + SX. \quad (10)$$

Thus,

$$\frac{\Delta Y}{\Delta E} = 1.05S, \quad (11)$$

where S represents the slope $\Delta Y/\Delta X$, and $\Delta Y/\Delta E$ represents the initial slope of the actual curves for $E = 0$.

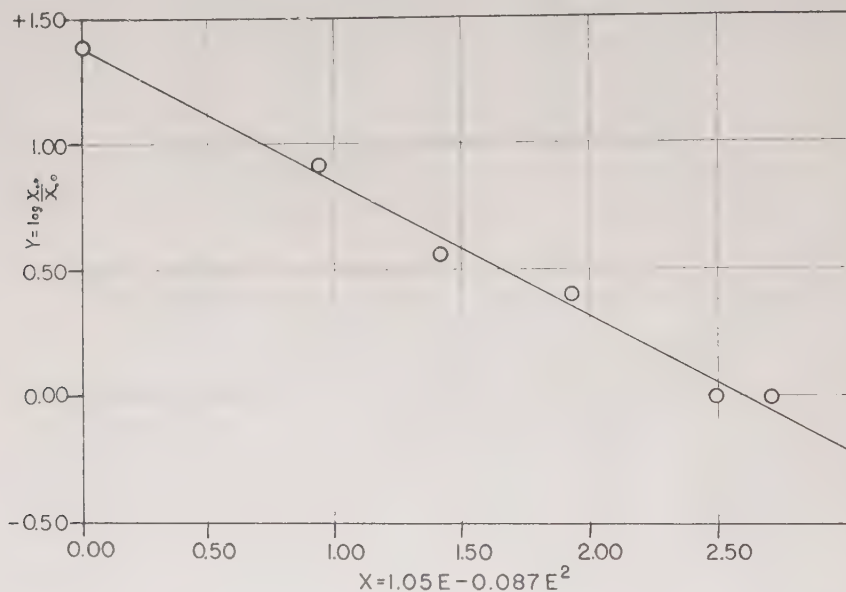


FIG. 6

When this procedure was followed for the iron lines in α Boötis, the points fell on a straight line with a slope $S = -0.520$, as shown in Figure 6. The stellar temperature was then derived by the simple equation

$$5040 \left(\frac{1}{T_{\odot}} - \frac{1}{T_*} \right) = 1.05S. \quad (12)$$

In Table 2 the atoms used for the determination of the slope are listed, and the values of the slope are given for both stars. Since all these values are relative to the curve of growth for the sun by Wright, his temperature determinations have been used. He gives 4900° for $Fe\ I$, 4550° for $Ti\ I$, and 4700° as the mean value for all other atoms. The weights ascribed to each element are based on the number of lines, the range in excitation potential, and the scatter of the individual points relative to the curves of growth. The second set of values given for $Ni\ I$ and $V\ I$ in 70 Ophiuchi was obtained from the differential values of giant and dwarf with the temperature already computed for α Boötis.

The scatter in the original relationship is exceptionally great, while the scatter for the second set is comparatively small. In obtaining the weighted mean temperature, both values were included. *

It follows that the intercept of the curve on the Y axis at E.P. = 0.00 will give $\log N_{0*}/N_{0\odot}$. That is, Y_0 yields the number of atoms of a given element in their normal state above equal areas of the photosphere in the star relative to the sun. We now make the approximation that Y_0 yields the relative number of all neutral atoms of a given kind. Table 3 lists the different atoms for which sufficient data are available to afford a deter-

TABLE 2

$$S = \frac{\Delta Y}{\Delta X} \text{ AND } S' = \frac{\Delta Y}{\Delta E} \text{ FOR VARIOUS ATOMS}$$

ELEMENT	α BOÖTIS			70 OPHIUCHI		
	Wt.	S	T	Wt.	S'	T
Ti I.....	5	-0.537	3010°	5	-0.141	4050°
Sc I.....	1	.540	3080	1	.184	4010
Cr I.....	1	.510	3130	1	.173	4050
Ni I.....	3	.506	3140	{ 2	.089	4350
Fe I.....	15	.520	3200	{ 3	.137	4160
V I.....	2	.480	3200	15	.180	4170
Co I.....	2	-0.426	3320	{ 1	.195	3980
Weighted means			3160	{ 2	.175	4040
				2	-0.089	4350
						4150

mination of Y_0 . In general, there are insufficient data for the enhanced atoms to permit an independent determination of slope. Thus the slopes of the respective neutral atoms have been applied for the extrapolation of Y_1 . The differences $Y_0 - Y_1$ have been tabulated for those atoms for which two stages of ionization are given.

D. DETERMINATION OF ELECTRON PRESSURES

By comparing the number of neutral and singly ionized atoms, it is possible to determine an approximate value of the electron pressure from the ionization equation:

$$\log \frac{N_1}{N_0} = \frac{-5040I}{T} + \frac{5}{2} \log T - 6.5 - \log P_e, \quad (13)$$

where I is the ionization potential, P_e the electron pressure, N_1/N_0 the ratio of the number of ionized to neutral atoms, and T the temperature. Strictly speaking, T should be the ionization temperature; but, in lieu of such data, the excitation temperatures have been used. If we now compare the number of neutral and ionized atoms of the same element in two stars, we have

$$\log \frac{P_{e*}}{P_{e\odot}} = \log \frac{N_{0*}}{N_{0\odot}} - \log \frac{N_{1*}}{N_{1\odot}} + \frac{5}{2} \log \frac{T_*}{T_\odot} + 5040I \left(\frac{1}{T_\odot} - \frac{1}{T_*} \right). \quad (14)$$

The first two quantities on the right-hand side of the equation are found in Table 3, and the remaining terms are evaluated from Table 2. The resulting values of the electron pressures for α Boötis and 70 Ophiuchi relative to the sun are found in Table 4.

V. DISCUSSION

A. BEHAVIOR OF CALCIUM, THE RARE EARTHS, AND CYANOGEN

The atoms of special interest in a comparative study of the giant and dwarf stars are *H*, *Na* I, *Ca* I, *Ca* II, *Sr* II, *Ba* II, *La* II, the singly ionized rare earths, and the *CN* and

TABLE 3
RELATIVE ABUNDANCES OF VARIOUS ATOMS

$$Y_0 = \log \frac{N_{0*}}{N_{0\odot}}, Y_1 = \log \frac{N_{1*}}{N_{1\odot}}$$

ELEMENT	α BOÖTIS		70 OPHIUCHI		DIFFERENCE
	$Y_{0,1}$	$Y_0 - Y_1$	$Y_{0,1}$	$Y_0 - Y_1$	
<i>Ca</i> I*	1.81		0.54		1.27
<i>Sc</i> I.....	1.93	+0.45	{.67	0.29	{1.26
<i>Sc</i> II.....	1.48		{.38}		{1.10
<i>Ti</i> I.....	2.51	+ .92	{.75	.30	{1.76
<i>Ti</i> II.....	1.59		{.45}		{1.14
<i>V</i> I.....	2.34	- .12	{.57	.00	{1.77
<i>V</i> II.....	2.46		{.57}		{1.89
<i>Cr</i> I.....	2.23	+ .64	{.83	.45	{1.40
<i>Cr</i> II.....	1.59		{.38}		{1.21
<i>Mn</i> I*	2.35		.62		1.73
<i>Fe</i> I.....	1.94	+ .70	{.53	.40	{1.41
<i>Fe</i> II.....	1.24		{.13}		{1.11
<i>Co</i> I.....	1.99		.43		1.56
<i>Ni</i> I.....	1.84		.28		1.56
<i>Zr</i> I*	2.06	+0.76	{.75	0.60	{1.31
<i>Zr</i> II.....	1.30		{.15}		{1.15
<i>La</i> II.....	1.11		.10		1.01
<i>Ce</i> II.....	1.13		.10		1.03
<i>Pr</i> II.....	1.48		.40		1.08
<i>Nd</i> II.....	1.76		.07		1.69
<i>Sm</i> II.....	1.29		.11		1.18
<i>Eu</i> II.....	1.66		0.00		1.66

* The slopes used for *Ca* I and *Mn* I in α Boötis were those determined by Adams and Russell, -0.62 and -0.70, respectively. For *Zr* I, *S* = -0.50 was assumed. The slopes assumed for these three atoms in 70 Ophiuchi were -0.20, -0.21, and -0.125.

TABLE 4
COMPUTED ELECTRON PRESSURES: $\log P_{e*}/P_{e\odot}$

Element	Wt.	α Boötis	70 Ophi- uchi	Differ- ence	Element	Wt.	α Boötis	70 Ophi- uchi	Differ- ence
<i>Ti</i>	2	-3.36	-0.78	-2.58	<i>Fe</i>	3	-4.04	-1.19	-2.85
<i>Cr</i>	1	3.39	0.84	2.55	Mean.....		-3.74	-1.10	-2.64
<i>Sc</i>	1	3.73	1.09	2.64					
<i>V</i>	1	-3.95	{ 1.50 -1.34	{ 2.45 -2.61					

CH molecules. In the present investigation, *Ca* I, *La* II, the rare earths, and *CN* are particularly noted.

Mention was made in Section III of the decrease in the intensity of the neutral calcium lines as we pass from dwarf to giant. Along the main sequence, the intensity of the lines increases very noticeably as we go to later spectral classes. The lower density in the atmosphere of the giant star must therefore more than compensate for the decrease in temperature. Miss Burwell found the $4^3P^o - 5^3S$ transition stronger in the dwarfs, and some members of the $3^3D - dp^3F^o$, $3^3D - 5^3P^o$, and $4^1P^o - p^2D$ multiplets slightly stronger in the dwarf. The strongest line in the $3^3D - dp^1D^o$ group, λ 6450, was observed to be stronger in the giant. In the present investigation, lines due to the $3^1D - 6^1P^o$, $4^1P^o - 6^1D$, $4^3P^o - 4^3D$, and $3^3D - 4^3F^o$ transitions were observed stronger in the dwarf. It is tempting to conclude that the apparent inconsistency of λ 6450 is due to the intersystem combination. However, two strong lines of the $4^3P^o - p^2P$ transition were observed strengthened in the giant in the present investigation, and the situation becomes puzzling.

The singly ionized rare earths and lanthanum are conspicuously strengthened in the giant. As yet, no detailed comparative study of these lines has been made in other giant and dwarf stars of late spectral type. It is thus difficult to know whether the observed strengthening is due to an absolute-magnitude effect or to a great abundance of these atoms in this giant star. It is interesting to note, however, that, with the exception of helium and hydrogen, calcium attains the greatest height for the neutral atoms in the solar atmosphere, while lanthanum and the rare earths are very low-lying. The apparent weakening of one atom and the strengthening of the others in the giant star might thus be intimately connected with an apparent stratification of the elements in stellar atmospheres.

The behavior of the *CN* band to the violet of λ 3883 in α Boötis is rather startling. The increase in the absorption of *CN* as we pass from dwarf to giant is one of the outstanding criteria for absolute-magnitude determination. The lines of the ultraviolet band are heavily blended, and the differences in the background intensities of the two stars in this region of the spectrum are great. The observed weakening of the lines in the giant might thus seem due to a large systematic error in the visual comparisons. However, lines of *Ti* I, *Fe* I, and weaker lines due to the rare earths in the same wave-length region were observed strengthened in the giant. No *CN* line in this region was observed to increase its strength in the spectrum of the giant. The abnormal weakness of the band at λ 4216 in α Boötis has already been mentioned in connection with the discrepancy of the trigonometric and spectroscopic parallax determinations, and the behavior of the λ 3883 band emphasizes this abnormality.

B. COMPARISON OF RESULTS FOR 70 OPHIUCHI AND SUNSPOTS

It is interesting to compare the results for temperature, pressure, and abundances in 70 Ophiuchi with those obtained by Mrs. Sitterly for the sunspot spectrum. A comparison of the slopes $S' = \Delta Y / \Delta E$ for *Fe* I and *Ti* I indicates an agreement within 30° for the two spectra, if we use the same solar value of the temperature. In the study of the stellar temperature, excitation temperatures were used, while in the study of the spot spectrum photospheric temperatures were used. The pressures agree within a factor of 8 if we use the mean of the individual temperature determinations for 70 Ophiuchi and within a factor of 3 if we use the mean value of the temperature for all the different atoms. However, the relative abundances disagree systematically. $\log N_{\text{spot}} / N_*$ is positive for all neutral atoms and negative for all ionized atoms. Some of the difference in the abundance is undoubtedly due to the difference in the characteristic layers of the two K0 spectra considered. In the spot spectrum we are going down to a greater optical depth than in the K0 stellar atmosphere. Some of the difference may, of course, be intrinsic, since there is a question whether one would expect entirely similar conditions for this localized K0 region in a G0 star as for the net spectrum of a K0 star.

C. EVIDENCE SUGGESTING A STRATIFICATION OF THE ELEMENTS

From Table 2 we see that the run in temperature for the different atoms in α Boötis and 70 Ophiuchi follows closely with the exception of $V\text{ I}$. Despite the uncertainty of the data for atoms of low weight, the sequence as a whole is quite similar to the sequence in the heights of the different atoms in the solar atmosphere given by Mitchell³⁵ with the high temperature corresponding to the low-lying element. On this basis, $V\text{ I}$ yields too high a temperature for α Boötis, and $Ni\text{ I}$ somewhat too low. The run in values of the electron pressures for the two stars also follows very closely, with the exception of $V\text{ I}$. However, this apparent consistency in electron pressures is to be expected when the individual values of the temperature are used. When the mean temperatures are used, i.e., 3160° for α Boötis and 4150° for 70 Ophiuchi, we obtain the values given in Table 5. It is noted

TABLE 5
COMPUTED ELECTRON PRESSURES, $\log P_{es}/P_{e\odot}$, BASED ON MEAN TEMPERATURES

Element	Wt.	α Boötis	70 Ophiuchi	Difference	Element	Wt.	α Boötis	70 Ophiuchi	Difference
<i>Ti</i>	2	-3.06	-0.77	-2.29	<i>V</i>	1	-4.09	-0.78	-3.31
<i>Cr</i>	1	3.31	.61	2.70					
<i>Sc</i>	1	3.41	.74	2.67	Mean.....		-3.55	-0.76	-2.79
<i>Fe</i>	3	-3.82	-.80	-3.02					

that the pressures are very similar for the different atoms in 70 Ophiuchi. However, the sequence for α Boötis is still very similar to that found in Table 4. We may say, then, that to a first approximation the behavior of the elements in the atmosphere of the giant suggests a sequence in stratification similar to that in the solar atmosphere. The behavior of $Ca\text{ I}$ and the rare earths supports this view. Furthermore, it is evident from the shape of the hydrogen lines and the strong double reversal of the $Ca\text{ II}$ lines that these elements are at high levels, as in the sun. Adams³⁶ studied the wave lengths of lines of different atoms in α Boötis and concluded:

The results found for the lines of different elements in Arcturus indicate a general arrangement of the gases in the star's atmosphere very similar to that in the sun. . . . We know that in the sun hydrogen gas rises to very great heights, and the gases of magnesium and calcium (leaving out of consideration in the latter element the H and K lines) also attain a high level, though somewhat lower than hydrogen. Similarly, titanium is a relatively high-level element, but iron is low. In Arcturus, we find the lines of all of these elements giving wave lengths shorter than those of iron, the greatest difference being for hydrogen, the next for magnesium and calcium, and the least for titanium. It seems reasonable, therefore, to conclude that the lines of these elements are subject to less pressure than are those of iron, and consequently that the gases producing them lie at a higher average level.

VI. CONCLUSION

The preceding analysis has shown that it is possible to obtain a large amount of valuable quantitative data from the weak lines in late-type stars by the use of estimated equivalent widths. For a study of the dwarf stars, the estimated values alone suffice for a preliminary investigation. For the giant stars, however, accurate equivalent widths of very strong lines (notably those of $Fe\text{ I}$) are necessary to fix the curve of growth. These values can be readily obtained from the direct-intensity tracings by Williams and Hiltner or from detailed microphotometric measures.

A comparison of the spectra of α Boötis and 70 Ophiuchi A in the photographic region has shown that:

³⁵ *Eclipses of the Sun*, p. 342, 4th ed., Columbia University Press, 1935.

³⁶ *Ap. J.*, **33**, 71, 1911; *Mt. W. Contr.*, No. 50.

APPENDIX

LINES DIFFERING IN INTENSITY BETWEEN α BOÖTIS AND 70 OPHIUCHI A
FOR WHICH NO IDENTIFICATION IS GIVEN
(WAVE-LENGTH REGION 3900-4808)

λ	Disk	Spot	Int. Diff.	Class	λ	Disk	Spot	Int. Diff.	Class
3899.828	0		2g		4181.977	2	2	2d	B
3901.600	2	2	3d	B	4184.000	4	3	1d	A
3908.276	1		1d?		4190.242	0	1	3g	A
3910.336	2	1	2d	A	4192.574	2	2	3d	B
3913.257	1	1	1d?	B	4194.091	-2	-1	1g	A
3914.015	1	0	1d	A	4207.409	1	0	2d	A
3915.614	3	2	1d	A	4212.978	-1		2g	
3917.36	0	0	1g	B	4216.602	1	1	3d	B
3921.189	1	1	3g	B	4220.808	-1		1g	
3946.555	0	0	1g	B	4221.174	-1		1g	
3959.838	1	1	1d?	B	4221.475	1	1	2d	B
3963.442	1	1	1d	B	4223.098	1	2	3g	A
3992.253	2	2	1d	B	4223.238	-1	0	2d	C
3993.472	-1		3g		4228.108	-2		1g?	
3998.475	0	0	2g	B	4228.318	-1		1g?	
4020.277	1	1	2d	B	4231.417	-2		1g	
4037.124	2	2	1d?	B	4244.249	0	-1	1d	A
4038.481	-1		1g		4245.086	0	-1	2d	A
4043.353	0	0	1d?	B	4253.006	1	1	2d	B
4048.402	-1		2g		4253.212	1	2	2d	C
4049.227	0	0	2g	B	4253.734	0	1	2d	C
4056.455	0	0	4g	B	4256.135	0	0	1g	B
4059.508	0	0	1d?	B	4256.608	0	0	2d	B
4092.001	-1	0	2g	A	4256.814	1	1	2d	B
4096.698	0	1	2g	A	4259.766	0	-1	2d	A
4097.462	-2	-2	2g	B	4322.833	-2	-1	1g	A
4099.056	0	1	3g	A	4324.414	2	2	2d	B
4102.623	-1	-1	1g	B	4326.922	0	-1	1g	C
4104.308	-1	0	2g	A	4328.612	0	0	2d	B
4105.830	-2	1	2g	A	4329.288	0	0	1d	B
4107.298	-1	0	2g	A	4335.275	1	1	1d	B
4114.120	-1	0	3g	A	4346.297	1	1	1d	B
4116.210	-2		2g		4396.067	-1		1d?	
4117.590	-1		3g		4396.155	-1			
4119.056	-2	0	2g	A	4396.435	-1	-1	1d?	B
4125.379	-2		2g		4396.963	1	0	1d?	A
4132.713	1	2	1g?	A	4398.495	0	-1	1d?	A
4134.900	-1	1	1g	A	4401.027	1	1	1d	B
4136.295	-1		2g		4403.189	1	1	1d	B
4141.313	-2		1g?		4406.163	0	0	2g	B
4144.384	-1		1g		4406.999	-1		2g	
4145.865	-1		2g		4412.704	-1		1d?	
4146.990	2	1	2g	C	4413.123	-1		1d?	
4147.214	-1	0	2g	A	4419.281	-1	-2	1d	A
4148.174	-1		2g		4420.288	1	1	2d	B
4148.396	-1	0	2g	A	4422.067	0	0	1d?	B
4148.921	-1	0	2g	A	4425.771	-1	-1	3g	B
4152.914	-1		3g		4430.770	0	-1	1d?	A
4155.055	-1	-1	1g	B	4431.04	-1	-1	1d?	B
4155.525	-1		2g		4431.850	0	0	2d	B
4162.122	-1		2g		4441.094	0	0	1d?	B
4162.464	1	0	1g	C	4480.828	0	0	1d	B
4162.910	-1	-1	1g	B	4499.145	1	1	2d	B
4164.787	0	1	3g	A	4509.744	1	1	1d	B
4169.618	1	1	1d	B	4518.344	1	1	3g	B
4170.147	-1	-1	1g?	B	4532.967	0	0		
4175.133	1	-1	3d	A	4533.053	0	0	2d	B

APPENDIX—Continued

λ	Disk	Spot	Int. Diff.	Class	λ	Disk	Spot	Int. Diff.	Class
4550.125	-1	-1	1d?	B	4658.304	0	0	1g?	B
4552.145	0	0	1d	B	4672.336	3	2	3d	A
4557.287	0	-1	2d	A	4673.791	-1		1d	
4561.419	1	1	1d	B	4674.101	1	1	1d	B
4567.585	-2		1g		4678.176	3	2	2d	A
4582.313	0	-1	2g	C	4688.690	0		2d	
4582.513	-1		1g		4691.601	1	0	2d	A
4584.276	-1		1g		4692.849	-1		1d?	
4585.348	0	-1	1d	A	4699.342	3	2	2d	A
4597.387	0	0	1d	B	4716.510	-2	0	1g	A
4603.352	0	1	1g	A	4716.838	-1	0	1g	A
4604.247	-1	0	1g	A	4717.129	-3		1g	
4605.596	2	2	1g	B	4751.555	-2		2g	
4640.294	1	0	1d	A	4771.472	3	2	1d?	A
4647.960	1	1	1d	B	4783.863	-2	-1	1g	A
4655.795	-1	-1	1g	B	4793.981	-1	-2	1g	C

1. The effects of absolute magnitude are marked for the atoms of *H*, *Na* I, *Ca* I, *Ca* II, *La* II, and the singly ionized rare earths.

2. The weakness of *CN* in α Boötis indicates an unusually low abundance of the molecule and possibly accounts for the anomalous parallax determinations of the star. The relative strengthening of the λ 3883 band of *CN* in the dwarf is startling and at present unexplained.

3. The curve of growth for α Boötis corresponds to an atomic velocity of 3.25 km/sec and a damping factor of $\Gamma/\gamma_0 = 0.8 \times 10^{-6}$.

4. Departure from thermodynamic equilibrium is observed in the spectrum of the giant. The empirical correction formulated by Adams and Russell fits the data satisfactorily.

5. The weighted mean excitation temperature and the electron pressure are 3160° and -3.74 for α Boötis and 4150° and -1.10 for 70 Ophiuchi.

6. The temperatures and electron pressures for the sunspot and 70 Ophiuchi A agree closely. The relative abundances of neutral and ionized atoms disagree systematically.

7. The behavior of several atoms, as well as the data for temperature and electron pressure, suggest a stratification of the elements similar to that in the solar atmosphere.

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³⁷ *Multiplet Table of Astrophysical Interest*, Princeton, 1945.